

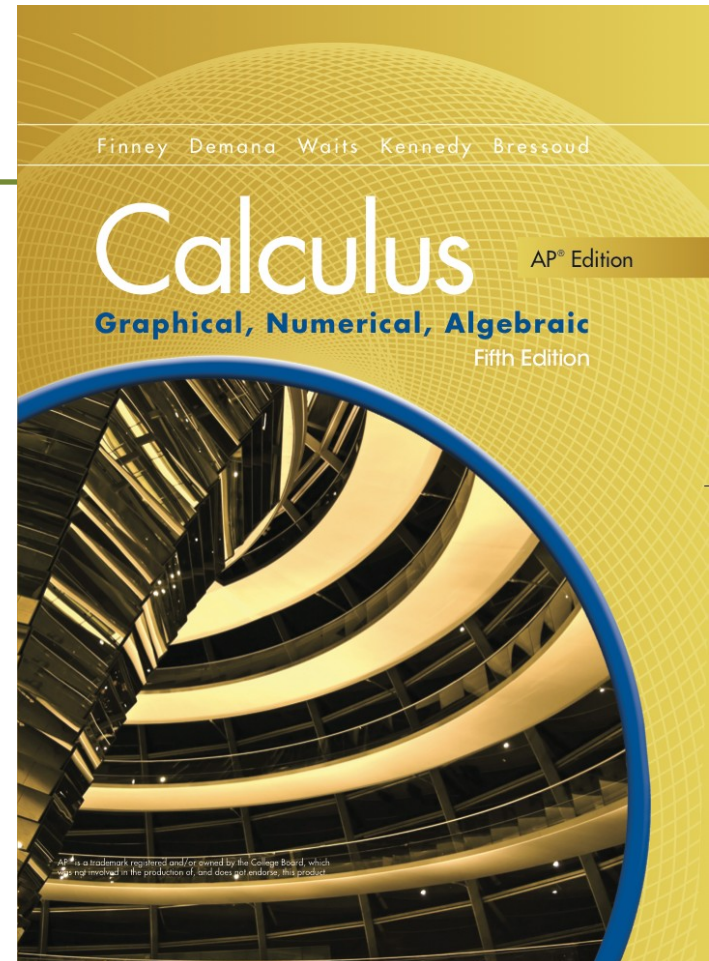
Chapter 1

Prerequisites for Calculus



Section 1.3

Exponential Functions



Quick Review

In Exercises 1- 3, evaluate the expression. Round your answers to 3 decimal places.

1. $5^{\frac{2}{3}}$

2. $3^{\sqrt{2}}$

3. $3^{1.5}$

In Exercises 4- 6, solve the equation. Round your answers to 4 decimal places.

4. $x^3 = 17$

5. $x^5 = 24$

6. $x^{10} = 1.4567$

Quick Review

In Exercises 7 and 8, find the value of investing P dollars for n years with the interest rate r compounded annually.

7. $P = \$500$, $r = 4.75\%$, $n = 5$ years

8. $P = 1000$, $r = 6.3\%$, $n = 3$ years

Quick Review

In Exercises 9 and 10, simplify the exponential expression.

$$9. \frac{(x^3 y^2)^2}{(x^4 y^3)^3}$$

$$10. \left(\frac{a^3 b^2}{c^4} \right)^2 \left(\frac{a^4 c^2}{b^3} \right)^{-1}$$

Quick Review

In Exercises 1- 3, evaluate the expression. Round your answers to 3 decimal places.

1. $5^{\frac{2}{3}}$

2.924

2. $3^{\sqrt{2}}$

4.729

3. $3^{1.5}$

0.192

In Exercises 4- 6, solve the equation. Round your answers to 4 decimal places.

4. $x^3 = 17$

2.5713

5. $x^5 = 24$

1.8882

6. $x^{10} = 1.4567$

± 1.0383

Quick Review

In Exercises 7 and 8, find the value of investing P dollars for n years with the interest rate r compounded annually.

7. $P = \$500$, $r = 4.75\%$, $n = 5$ years **\$630.58**
8. $P = 1000$, $r = 6.3\%$, $n = 3$ years **\$1201.16**

Quick Review

In Exercises 9 and 10, simplify the exponential expression.

$$9. \frac{(x^3 y^2)^2}{(x^4 y^3)^3}$$

$$\frac{1}{x^{18} y^5}$$

$$10. \left(\frac{a^3 b^2}{c^4} \right)^2 \left(\frac{a^4 c^2}{b^3} \right)^{-1}$$

$$\frac{a^2}{bc^6}$$

Quick Review Solutions

In Exercises 1- 3, evaluate the expression. Round your answers to 3 decimal places.

1. $5^{\frac{2}{3}}$	2.924	2. $3^{\sqrt{2}}$	4.729
3. $3^{1.5}$	0.192		

In Exercises 4- 6, solve the equation. Round your answers to 4 decimal places.

4. $x^3 = 17$	2.5713	5. $x^5 = 24$	1.8882
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Quick Review Solutions

In Exercises 7 and 8, find the value of investing P dollars for n years with the interest rate r compounded annually.

$$7. P=\$500, \quad r=4.75\%, \quad n=5 \text{ years} \quad \$630.58$$

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In Exercises 9 and 10, simplify the exponential expression.

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$$10. \left(\frac{a^3 b^2}{c^4} \right)^2 \left(\frac{a^4 c^2}{b^3} \right)^{-1} \quad \frac{a^2}{bc^6}$$

What you'll learn about

- Exponential functions
- Rules of exponents
- Applications of exponential functions (growth and decay)
- Compound interest
- The number e

...and why

Exponential functions model many growth patterns.

Exponential Function

Let a be a positive real number other than 1.

The function

$$f(x) = a^x$$

is the **exponential function with base a** .

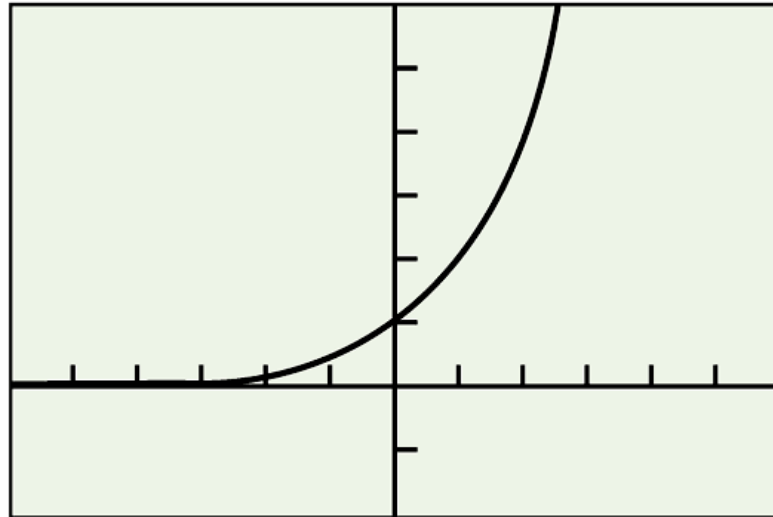
The domain of $f(x) = a^x$ is $(-\infty, \infty)$ and the range is $(0, \infty)$.

Compound interest investment and population growth are examples of exponential growth.

Exponential Growth

If $a > 1$ the graph of f looks like the graph of $y = 2^x$.

$$y = 2^x$$



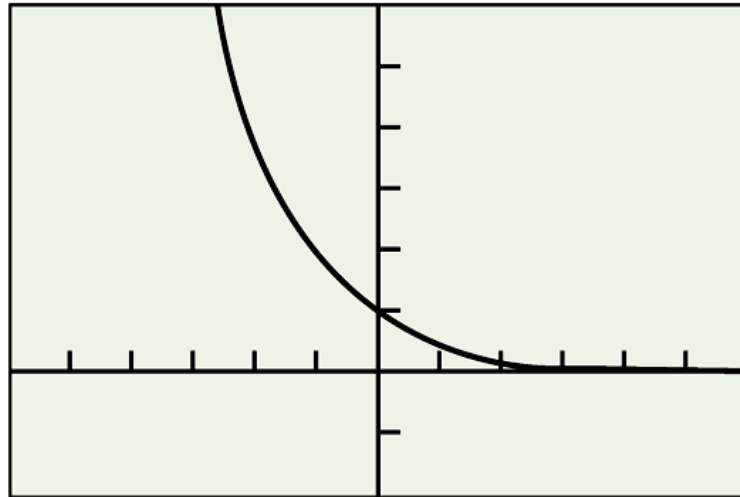
$[-6, 6]$ by $[-2, 6]$

(a)

Exponential Growth

If $0 < a < 1$ the graph of f looks like the graph of $y = 2^{-x}$.

$$y = 2^{-x}$$



$[-6, 6]$ by $[-2, 6]$

(b)

Rules for Exponents

If $a > 0$ and $b > 0$, the following hold for all real numbers x and y .

$$1. a^x \cdot a^y = a^{x+y}$$

$$4. a^x \cdot b^x = (ab)^x$$

$$2. \frac{a^x}{a^y} = a^{x-y}$$

$$5. \left(\frac{a}{b} \right)^x = \frac{a^x}{b^x}$$

$$3. (a^x)^y = (a^y)^x = a^{xy}$$

Half-life

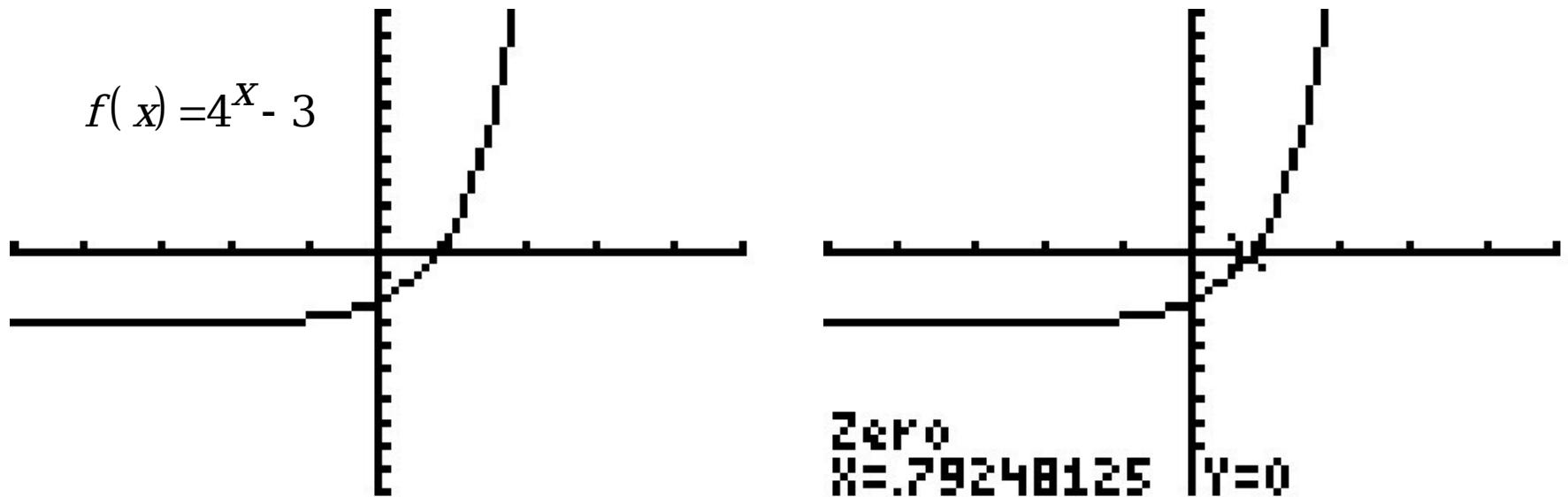
Exponential functions can also model phenomena that produce decrease over time, such as happens with radioactive decay. The **half-life** of a radioactive substance is the amount of time it takes for half of the substance to change from its original radioactive state to a non-radioactive state by emitting energy in the form of radiation.

Exponential Growth and Exponential Decay

The function $y=k \cdot a^x$, $k>0$, is a model for **exponential growth** if $a>1$, and a model for **exponential decay** if $0 < a < 1$.

Example Finding Zeros

Use a grapher to find the zero's of $f(x) = 4^x - 3$.



$[-5, 5]$ $[-10, 10]$

Compound Interest Formula

If an account begins with principal P and earns an annual interest rate r compounded n times per year, then the value of the account in t years is

$$P \left(1 + \frac{r}{n} \right)^{nt} .$$

The Number e

Many natural, physical and economic phenomena are best modeled by an exponential function whose base is the famous number e , which is 2.718281828 to nine decimal places.

We can define e to be the number that the function

$f(x) = \left(1 + \frac{1}{x}\right)^x$ approaches as x approaches infinity.

The Number e

The exponential functions $y=e^x$ and $y=e^{-x}$ are frequently used as models of exponential growth or decay.

Example The Number e

The approximate number of fruit flies in an experimental population after t hours is given by

$$Q(t) = 20e^{0.03t}, \quad t \geq 0.$$

- Find the initial number of fruit flies in the population.
- How large is the population of fruit flies after 72 hours?
- Use a grapher to graph the function Q .

Example The Number e

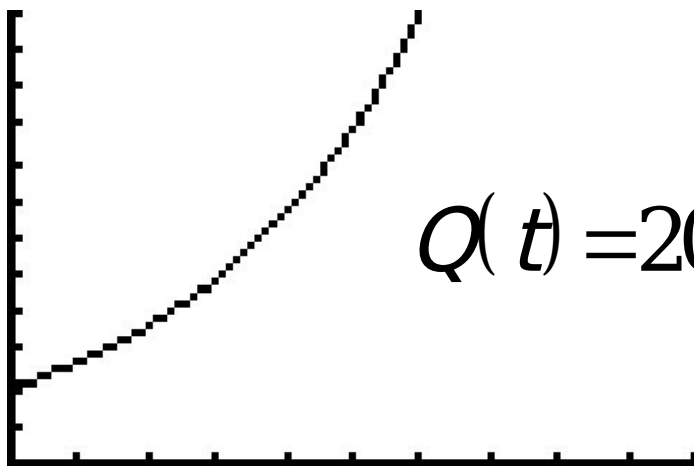
a. To find the initial population, evaluate $Q(t)$ at $t=0$.

$$Q(0) = 20e^{0.03(0)} = 20e^0 = 20(1) = 20 \text{ flies.}$$

b. After 72 hours, the population size is

$$Q(72) = 20e^{0.03(72)} = 20e^{2.16} \approx 173 \text{ flies.}$$

c.



Continuously Compounded Interest Formula

If an account begins with principal P and earns an annual interest rate r compounded continuously, then the value of the account in t years is

$$P(e^r)^t = Pe^{rt}.$$

Quick Quiz Sections 1.1 – 1.3

You may use a graphing calculator to solve the following problems.

1. Which of the following gives an equation for the line through $(3, -1)$ and parallel to the line: $y = -2x + 1$?

(A) $y = \frac{1}{2}x + \frac{7}{2}$

(C) $y = -2x + 5$

(B) $y = \frac{1}{2}x - \frac{5}{2}$

(D) $y = -2x - 7$

(E) $y = -2x + 1$

Quick Quiz Sections 1.1 – 1.3

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1. Which of the following gives an equation for the line through $(3, -1)$ and parallel to the line: $y = -2x + 1$?

(A) $y = \frac{1}{2}x + \frac{7}{2}$

(C) $y = 2x + 5$

(B) $y = \frac{1}{2}x - \frac{5}{2}$

(D) $y = -2x - 7$

(E) $y = -2x + 1$

Quick Quiz Sections 1.1 – 1.3

2. If $f(x) = x^2 + 1$ and $g(x) = 2x - 1$, which of the following gives $(f \circ g)(2)$?

- (A) 2
- (B) 5
- (C) 9
- (D) 10
- (E) 15

Quick Quiz Sections 1.1 – 1.3

2. If $f(x) = x^2 + 1$ and $g(x) = 2x - 1$, which of the following gives $(f \circ g)(2)$?

- (A) 2
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- (C) 9
- (D) 10
- (E) 15

Quick Quiz Sections 1.1 – 1.3

3. The half-life of a certain radioactive substance is 8 hours. There are 5 grams present initially. Which of the following gives the best approximation when there will be 1 gram remaining?

- (A) 2
- (B) 10
- (C) 15
- (D) 16
- (E) 19

Quick Quiz Sections 1.1 – 1.3

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- (A) 2
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- (C) 15
- (D) 16
- (E) 19